Discovering the Area of a Circle: Egyptian Style

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What a grandiose goal claimed by the author of the Rhind Papyrus, one of the oldest mathematical manuscripts known to man. History of mathematics often provides motivation and interesting stories that help students to master mathematics. The Rhind Papyrus was purchased at a market by a 25 year old Scotsman, Alexander Henry Rhind, visiting Egypt in 1858. Rhind died a few years later and the papyrus journeyed to the British Museum where it remains today. The introduction of the work tells us that it was written by the scribe Ahmes in 1650 BCE, more than three thousand years ago; however, Ahmes also states that it is not his original work, but rather a copy of a papyrus created more than 200 years earlier. This mathematical document provides insight into the mathematics done in ancient Egypt. The format is the statement of a problem followed by its solution, and is, perhaps, the first teacher's edition of any mathematical text!

"Grasping the meaning of things and knowing everything that is." Ahmes

The papyrus provides an opportunity to flavor today's classrooms with a different perspective on the traditional problem of finding the area of a circle. In this article we will carefully examine problem #50 from the Rhind Papyrus that demonstrates the Egyptians' understanding of the computation of the area of a circle.

In middle school, students learn the age old formula for the exact area of a circle, $A = \pi r^2$. For example the area of a circle with radius $\frac{9}{2}$ units is equal to $\pi\left(\frac{9}{2}\right)^2$ square units. The area of the circle, correctly approximated to three decimals, is 63.617. The following problem from the Rhind Papyrus illustrates an unusual technique for estimating the area of this circle. The scribe explains a procedure that leads to an approximation of 64 square units, quite close to the exact area. Let us examine this process.

Problem 50 reads "Example of a round field of diameter 9. What is the area? Take away $\frac{1}{9}$ of the diameter; the remainder is 8. Multiply 8 times 8, it makes 64. Therefore, the area is 64." In the following table we interpret each step.

"Example of a round field of diameter 9."	Consider a circle of diameter 9 units.
"Take away $\frac{1}{9}$ of the diameter, the remainder is 8."	$\frac{1}{9}$ of the diameter is $\frac{1}{9}\cdot 9$. Subtract $\frac{1}{9}$ of the diameter from the diameter. 8 units remain. $9-\frac{1}{9}\cdot 9=\frac{8}{9}\cdot 9=8$
"Multiply 8 times 8, it makes 64."	$8\cdot 8=64$
"Therefore, the area is 64."	The area of the circle is 64 square units.

The Egyptians are defining a method for finding the area of a circle of diameter 9. How does their process compare to using the formula $A=\pi r^2$ for this circle? How did the Egyptians approximate π? The following procedure enables us to make the comparison. To begin, we use algebraic properties to rewrite the formula for the area of a circle in terms of the diameter.

In the formula for the area of a circle, $A=\pi r^2$, replace r, the radius, with $\frac{d}{2}$, the diameter divided by 2.	$A=\pi r^2$ $=\pi\left(\frac{d}{2}\right)^2$
Rewrite the expression as a multiple of d^2.	$=\pi\frac{d^2}{4}$ $=\frac{\pi}{4}d^2$ Thus, $A=\frac{\pi}{4}d^2$
Using this version of the formula, we find the area of a circle with diameter 9.	$A=\frac{\pi}{4}\cdot 9^2$ $=\frac{\pi}{4}\cdot 81$ $=\frac{81}{4}\cdot\pi$

According to the Egyptians when the diameter is equal to 9, then the area is equal to 64. Using our area formula we obtain the following sequence of equalities.	$64 = \frac{\pi}{4} \cdot 9^2$ $= \frac{\pi}{4} \cdot 81$ $= \frac{81}{4} \cdot \pi$ Thus, $64 = \pi \frac{81}{4}$
By solving the last equation for π, we find the Egyptian's approximation for the number π.	$\pi = 64 \cdot \frac{4}{81} \cdot 3.16$

It appears that many thousands of years ago, the Egyptians were approximating π by 3.16. Rather amazing, isn't it?

The discovery project[1] that follows provides a hands on activity to enable students to actively engage in the process described by the Egyptians and use that to approximate the value of π. The diameter of a penny is defined to be one unit. The students use pennies to measure the side of an 8 by 8 square, and then compute the area of the square.[2] Next, the students use pennies to measure the diameter of a circle with diameter 9 units, and then proceed to fill the circle with pennies to approximate the area of the circle. The students mimic the process explained by the scribe and find an approximation to the area of the circle, and thus, an approximation to π. What better way to capture students' attention than to provide a hands-on experience.

Please see page 54 of the Activity Section for the activity related to this article.

[1] The idea for this activity is based on an exercise in Victor Katz's book on the history of mathematics.
[2] Although the Egyptians were known to use counters in their mathematics, it is not known that the solution to this problem was obtained through their use.